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# AN EVOLUTIONARY APPROACH TO FRACTIONAL BILEVEL PROGRAMMING

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## ABSTRACT

This study introduces an innovative method that employs evolutionary computation to address interval-valued fractional bilevel programming (IVFBLP) problems. These involve decision-making at two hierarchical levels, where some data is expressed as intervals rather than fixed values, enhancing realism but increasing complexity. The proposed method integrates mixed 0–1 programming, goal programming, and genetic algorithms to efficiently address these challenges.

The genetic algorithm mimics natural evolutionary processes by generating multiple solution candidates and iteratively selecting the best, allowing the model to explore a wide solution space and converge towards optimal or near-optimal decisions. To ensure solutions closely meet desired goals while minimizing adverse outcomes, termed regrets, the approach employs two strategies—*minsum* and *minmax*—within a combined success-measuring function.

The solution process follows two phases: first, it establishes optimal target intervals representing achievable goal ranges; second, it identifies the best decisions for both leader (upper) and follower (lower) roles. This structured approach delineates responsibilities between decision-makers.

Utilizing evolutionary computing enables the model to handle uncertainties, nonlinearities, and complex structures effectively. The paper demonstrates practical applicability through a numerical example, showcasing its capability to solve real-world problems involving interval-based objectives and constraints where decisions occur at multiple hierarchical levels.

**Keywords :** Fractional bilevel programming, Goal programming, Evolutionary algorithm, Interval programming, Interval-valued fractional bilevel programming, Multiobjective decision-making.

## 1. INTRODUCTION

Bilevel programming (BLP) has been extensively studied as a mathematical framework for modelling decision problems with a hierarchical structure, particularly since the pioneering work of Candler and Townsley [4], who demonstrated its applicability in large-scale organizational planning and decision-making structures. A typical bilevel programming problem (BLPP) involves a pair of decision makers (DMs) operating at separate hierarchical stages—each directing their own decision variables and independently pursuing their respective objectives. However, in practical scenarios, it has become evident that mutual cooperation and willingness to compromise are essential for maintaining the organization's existence and fostering sustainable progress. This realization has led to a growing interest in cooperative decision-making models that balance individual and collective interests.

In this context, both BLPPs and their generalizations—multilevel programming problems (MLPPs)—have been investigated in depth [2,3,5,8, 22]. Additionally, fuzzy programming

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(FP) methods [7. 19] have been introduced as a means of dealing with uncertainty in decentralized decision-making. These methods have been successfully employed in real-world contexts, including traffic regulation, economic system planning, strategic defense, network design, and conflict management.

Research demonstrates that genetic algorithms (GAs), which serve as powerful methods for MODM problems [15, 16], have been adapted for BLPPs [12,18]. Additionally, the GA-based fuzzy goal programming (FGP) frameworks proposed by Pal et al. [29,30] for linear and fractional BLPPs and MLPPs indicate progress, though this field is still emerging.

Despite the success of FP and FGP approaches in MODM applications, one recurring challenge persists: assigning fuzzy aspiration levels to objectives, particularly when the underlying data or targets are ambiguous or ill-defined. This issue often arises in real-world decision-making environments characterized by uncertainty and imprecision.

To address this difficulty, interval programming (IP) has emerged as a valuable alternative. IP methods handle uncertainty by representing parameters as intervals rather than exact or fuzzy values. The methodological foundations of IP have been extensively studied by Olivera et al. [25] and further explored by Pal et al. [26,27]. A GA analyzed method to fractional IP problems has been proposed in [26], and its practical applicability has been demonstrated in [31]. However, the methodological development of IP remains at a nascent stage, and its integration with hierarchical decision-making models is still relatively unexplored in scholarly work.

Here, we examine a fractional bilevel programming problem characterized by interval-valued objective coefficients at both decision levels. Within a goal programming (GP) framework, we first determine target intervals and the control vector for the upper-level decision maker (leader) which is derived by evaluating the best and worst objective outcomes for both the leader and the lower-level decision maker (follower), implementing a GA framework

The objectives and control vector expressed as intervals are later reformulated into standard goals through interval arithmetic techniques. A goal achievement function is constructed to minimize both underachievement and overachievement variables related to the targeted goals, utilizing both *minsum* [11] and *minmax* [1] strategies. By focusing on reducing the lower thresholds of regret intervals corresponding to the intervals of goal, this method aims to derive a compromise solution that reconciles the interests of both DMs and contributes to the collective advantage of the organization.

To tackle the combinatorial complexity of the model, it is reformulated as a mixed 0–1 goal programming problem. A GA employed search approach is then employed to identify a satisfactory decision based on the relative importance assigned to goal achievements.

The suggested method is demonstrated through a mathematical example to demonstrate its practical utility and effectiveness.

## 2. Related Problem Formulation

Let's consider a vector $X=(x_1,x_2,\ldots,x_n)$ that contains all the decision variables for the two-level (hierarchical) decision-making systems.

For the k-th objective, $F_k$ represents the objective function (what we want to achieve), and $X_k$ represents the function that outlines the intended objective k=1,2; where $\bigcup_k \{X_k \mid k=1,2\} = X$..

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In a structured, multi-level decision environment, the fractional bilevel programming problem (BLPP) with interval-based coefficients can be written as:

Find the variables $X_1$ and $X_2$ so that the objectives are achieved.

$$\underset{X_1}{\text{Max}} F_1(X_1,X_2) = \frac{[c_{11}^L,c_{11}^U]X_1 + [c_{12}^L,c_{12}^U]X_2 + [\alpha_1^L,\alpha_1^U]}{[d_{11}^L,d_{11}^U]X_1 + [d_{12}^L,d_{12}^U]X_2 + [\beta_1^L,\beta_1^U]} \quad \text{(Leader's problem)} \tag{1}$$

and, for a specified $X_1, X_2$ solves

$$\underset{X_2}{\text{Max}} F_2(X_1,X_2) = \frac{[c_{21}^L,c_{21}^U]X_1 + [c_{22}^L,c_{22}^U]X_2 + [\alpha_2^L,\alpha_2^U]}{[d_{21}^L,d_{21}^U]X_1 + [d_{22}^L,d_{22}^U]X_2 + [\beta_2^L,\beta_2^U]} \quad \text{(Follower's problem)} \tag{2}$$

subject to,

$$X \in S = \left\{ (X_1,X_2) \mid A_1X_1 + A_2X_2 \begin{pmatrix} \geq \\ \leq \end{pmatrix} b,\ X \geq 0 \right\}, \tag{3}$$

Here, for k=1,2 the given terms are:

- **Interval coefficient vectors** $[c_{k\ell}^L, c_{k\ell}^U]$, $[d_{k\ell}^L, d_{k\ell}^U]$— values defined within a lower limit (L) and an upper limit (U), representing uncertain or variable data.
- **Constants** $\alpha_k^L, \alpha_k^U, \beta_k^L, \beta_k^U$— fixed numerical values.
- **Constant matrices** $A_1$ and $A_2$— tables of fixed numbers.
- **b** — a fixed vector.

It is also assumed that the collection of all solutions (the feasible region) $S(\neq \Phi)$ constitutes a **convex and continuous** shape, meaning any line between two points in the region stays entirely within it.

It is again generally accepted that $[d_{k1}^L, d_{k1}^U]X_1 + [d_{k2}^L, d_{k2}^U]X_2 + [\beta_k^L, \beta_k^U] > 0$.

Applying the interval arithmetic operation rule from [14], the interval-valued objectives in (1) and (2) have the capability to subsequently be represented as [27]:

$$\underset{X_1}{\text{Max}} F_1(X_1,X_2) = \left[ \frac{c_{11}^L X_1 + c_{12}^L X_2 + \alpha_1^L}{d_{11}^U X_1 + d_{12}^U X_2 + \beta_1^U}, \frac{c_{11}^U X_1 + c_{12}^U X_2 + \alpha_1^U}{d_{11}^L X_1 + d_{12}^L X_2 + \beta_1^L} \right] = [T_{1L}(X_1,X_2), T_{1U}(X_1,X_2)],\ \text{(say)} \tag{4}$$

$$\underset{X_2}{\text{Max}} F_2(X_1,X_2) = \left[ \frac{c_{21}^L X_1 + c_{22}^L X_2 + \alpha_2^L}{d_{21}^U X_1 + d_{22}^U X_2 + \beta_2^U}, \frac{c_{21}^U X_1 + c_{22}^U X_2 + \alpha_2^U}{d_{21}^L X_1 + d_{22}^L X_2 + \beta_2^L} \right] = [T_{2L}(X_1,X_2), T_{2U}(X_1,X_2)],\ \text{(say)} \tag{5}$$

In solving the problem, the GA approach is applied to identify the equations that demonstrate the intervals of focus (4) and (5) and to formulate the corresponding GP model. The procedure is elaborated in Section 3.

## 3. GA algorithm development

The Genetic Algorithm (GA) framework involves two key operational steps: selection and crossover. In the current GA search strategy, the selection mechanism is based on fitter codon selection [23], and the crossover mechanism employs a two-point crossover technique [9], as described below:

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**(i) Fitter Codon Selection:**

In a GA, chromosomes are typically represented as binary strings, with codons being segments of these strings. Traditional GA methods for optimization, such as those in [9,13,17], often employ the roulette-wheel selection technique [9] for choosing parent chromosomes. In contrast, the fitter codon selection method proposed in [23,28] involves comparing codons based on predefined string lengths to identify the better candidate. For the GA implementation, selection is performed by examining only a segment of the string—from the most significant bit to a specified length—without requiring evaluation of the full string or conversion to its numeric binary value. This selective focus significantly reduces computational overhead during the selection phase.

**(ii) Two-Point Crossover:**

While conventional GA approaches often utilize a single-point crossover strategy [13], the current GA model adopts a two-point crossover method as described in [9]. The key advantage of this approach is its skill in producing a more diverse new population from the initial one within fewer iterations compared to single-point crossover, thereby enhancing convergence efficiency.

The detailed algorithmic procedure is provided in Section 3.1.

**3.1 Outline of the Proposed GA Method**
**Step 1: Encoding and Initialization**

Let $V_P$ signifies the binary-coded version of a chromosome found in a population as $V_P = \{x_1, x_2, ..., x_n\}_P$, where 'n' with 'n' indicating the chromosome's length, P = 1, 2,..., pop_size, represents the population size, and where pop_size chromosomes are randomly initialized in its search domain.

**Step 2**. Fitness function

The fitness score for every chromosome is derived from the objective function. The fitness function is defined as

eval $(V_P) = (F_K)_P$, $\quad k = 1, 2;\ P = 1,2, . . ., pop\_size.$

The chromosomes associated with the maximum and minimum values of the objective function are identified as

$V^* = \max \{eval\ (V_P) \mid P = 1, 2,..., pop\_size\}$,

and $V^* = \min\{eval\ (V_P) \mid P = 1, 2, ..., pop\_size\}$, respectively.

**Step 3**. **Survivor Determination**

Within the suggested genetic algorithm (GA), a "fitter codon selection" method is used. This means chromosomes (possible solutions) are selected from the population based on how good (fit or optimal) they are in solving the problem. The advantage of this method is that it helps efficiently guide the search process toward a solution with a certain desired fitness level, improving convergence speed and solution quality.

For example, consider these four chromosomes in the population:
(i) 111010000
(ii) 111101010

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(iii) 010111010
(iv) 101010010

In this method, codons (bit segments within the chromosome) are compared based on how often the most important bits match at the beginning. The "codon length" begins at the most significant bit and proceeding up to the first differing bit, capturing early similarity. In the example, chromosomes (i) and (ii) have a codon length of 4, making them fitter than (iii) and (iv) due to a longer matching prefix. Between (i) and (ii), chromosome (ii) is even fitter than (i) because it continues to match key bits further. Importantly, we don't need to convert these binary strings into decimal numbers for comparison — we decide relative fitness directly from structural bit patterns, simplifying the evaluation process.

**Step 4**. Crossover

The chance that crossover will happen when two chromosomes reproduce is given by a value called $P_c$. In a two-point crossover, two parent chromosomes swap the middle part of their genes with each other to create new offspring. To pick a chromosome as a parent, two random numbers (r and $r_1$), each between 0 and 1, are chosen. Both r, $r_1 \in [0, 1]$ must be less than $P_c$, and when you add them together, they should be less than 1. For choosing two parents, a third number $r_2$ is also defined so that such that $r_2$=1-r- $r_1$.

**Step 5**. Mutation

$P_m$ is a value that shows how likely a mutation is to happen. Mutation is done one bit at a time, and for each bit, upon selecting a random number in the interval [0, 1], if it is less than $P_m$, that bit in the chromosome will be mutated.

**Step 6.** Termination

The genetic algorithm stops running after it has completed a set number of generations. At this point, the best chromosome that has been produced so far is chosen as the solution.

This top-performing chromosome is considered the result of the search by the algorithm. The decision made by the algorithm is based on the quality of this best chromosome. In summary, the process ends when the best possible answer is found within the given generations.

## 4. INTERVAL-BASED REPRESENTATION OF THE OBJECTIVE

To formulate the GP model specifically for the BLPP, you need to set the target ranges for both goals, $F_1$ and $F_2$. You also have to decide on the decision variables, called $X_1$ that the leader will manage. These details must be clearly established within the decision-making process. This helps guide how the model works and finds solutions.

### 4.1. Computing Intervals of Target

A target range is determined by first identifying the optimal and least favorable outcomes for each objective. This is accomplished by configuring the Genetic Algorithm with suitable parameters, allowing it to explore possible solutions. The leader's best and worst solutions then establish the boundaries for these target intervals.

Assume the leader's highest and lowest solutions for each objective are as follows.

$(X_1^{\ell b}, X_2^{\ell b}; T_{1U}^*)$ and $(X_1^{\ell w}, X_2^{\ell w}; T_{1L}^*)$ , respectively,

where $T_{1U}^* = \underset{(X_1,X_2)\in S}{Max}\ T_{1U}(X_1, X_2)$ , and $\quad T_{1L}^{*} = \underset{(X_1,X_2)\in S}{\text{Min}} T_{1L}(X_1, X_2)$.

Likewise, the optimal and least favorable solutions of the follower can be derived as

$(X_1^{fb}, X_2^{fb}; T_{2U}^{*})$ and $(X_1^{fw}, X_2^{fw}; T_{2L}^{*})$ , respectively,

where $\; T_{2U}^{*} = \underset{(X_1,X_2)\in S}{\text{Max}} T_{2U}(X_1, X_2)$,

and $\quad T_{2L}^{*} = \underset{(X_1,X_2)\in S}{\text{Min}} T_{2L}(X_1, X_2)$.

In the process for arriving into decisions, it is fairly assumed that both the hierarchical levels want to work in accordance. Each is ready to give up some of their own advantage to help the other. This partnership is regarded as crucial not only for ensuring their survival but also for securing the organization's future success.

From this standpoint, the target intervals corresponding to objective $F_k$ are found as

$$\left[t_k^L, t_k^U\right], \; k = 1,2$$

where $T_{kL}^{*} \le t_k^L \le t_k^U \le T_{kU}^{*}$, $k = 1,2$, and this depends on the process of decision horizon..

Accordingly, the formulations in (4) and (5) with their respective ranges of target may be expressed as:

$$\left[T_{1L}(X_1, X_2), T_{1U}(X_1, X_2)\right] = [t_1^L, t_1^U], \quad \text{(Leader's problem)} \qquad (6)$$

$$\left[T_{2L}(X_1, X_2), T_{2U}(X_1, X_2)\right] = [t_2^L, t_2^U], \quad \text{(Follower's problem)} \qquad (7)$$

Since the leader holds greater decision-making authority, they should allow some flexibility in their best decision, represented by $X_1^{lb}$, by relaxing it to a certain extent as a lower tolerance limit $X_1^{l}(X_1^{lw} < X_1^{l} < X_1^{lb})$. This relaxation helps create space for the follower to explore and find a better decision.

According to the principle of midpoint arithmetic in IP , the interval objective of the control vector $X_1$ can be expressed as

$$X_1 = [X_1^{l}, X_1^{lb}] \qquad (8)$$

**4.2. Precise goal modeling for objectives defined by intervals**

For developing the GP process of the study, the objectives listed in (6), (7), and (8) need to be converted into basic objective formats. This is achieved by defining target ranges and adding variables that represent deviations below and above these targets for each objective.

The standard way to represent the goals of the objectives can be formulated as

$$T_{1L}(X_1, X_2) + d_{1L}^{-} - d_{1L}^{+} = t_1^L, \qquad (9)$$

and $T_{1U}(X_1, X_2) + d_{1U}^{-} - d_{1U}^{+} = t_1^U$; (Leader's problem) (10)

$$T_{2L}(X_1, X_2) + d_{2L}^{-} - d_{2L}^{+} = t_2^L, \qquad (11)$$

and $T_{2U}(X_1, X_2) + d_{2U}^- - d_{2U}^+ = t_2^U$, (Follower's problem) (12)

where $(d_{kL}^-, d_{kU}^-) \geq 0,\ k = 1,2$ express the extent of deviation below the desired level, and $(d_{kL}^+, d_{kU}^+) \geq 0,\ k = 1,2$ indicate the over-deviational variables linked with the associated goal expressions.

In the same manner, the objective formulations for the vector of control $X_1$ are expressed as:

$$X_1 + d_L^- - d_L^+ = X_1^{\ell},$$

(13)

$$\text{and } X_1 + d_U^- - d_U^+ = X_1^{fb}. \quad (14)$$

where, $(d_L^-, d_L^+)$ and $(d_U^-, d_U^+) \geq 0$

correspond to the deviation vectors (under and over), the size of which is dependent on $X_1$.

## 5. MATHEMATICAL FORMULATION OF THE GP MODEL

In a process of decision analysis, each decision maker (DM) aims to reach their goal values within given target ranges by reducing the regrets, which are measured using deviational variables related to the decisions [32]. The function representing goal attainment is known as the regret function because it focuses on minimizing these regret intervals as much as possible within the decision-making environment. In interval programming, both the *minsu*m GP approach [11], which minimizes the total weighted unwanted deviations, and the *minmax* GP approach [1], which minimizes the largest deviation, are combined. This combination helps find a balanced and satisfactory solution that meets the target intervals for the goals.

From the hopeful perspective of both DMs,, the focus is on reducing the unavoidable regrets within the defined intervals of regret $(d_{kL}^-, d_{kU}^+), (d_{kL}^+, d_{kU}^-), (k = 1,2)$, and $(d_L^-, d_U^+), (d_L^+, d_U^-)$ are taken into consideration.

To simplify the model, assume that that $n_1 (n_1 < n)$ is the set size of variables of decision corresponding to the vector of control $X_1$.

Accordingly, the stated model can be represented as identifying $X(X_1,X_2)$ in order to

The problem reduces to minimizing Z=

$$\lambda\left\{\sum_{i=1}^{n_1+2}(w_{iL}^- d_{iL}^- + w_{iU}^+ d_{iU}^+) \wedge \sum_{i=1}^{n_1+2}(w_{iL}^+ d_{iL}^+ + w_{iU}^- d_{iU}^-)\right\} + (1-\lambda)\left\{\max_{i \in n_1+2}\{d_{iL}^- + d_{iU}^+\} \wedge (d_{iL}^+ + d_{iU}^-)\right\}$$

(15)

and must comply with the constraints of goals given in (9)–(14), under the system constraint described in (3),

Where Z represents the regret function, which quantifies the extent of goal achievement or the shortfall in meeting the goal.

$d_{iL}^-, d_{iU}^+, d_{iL}^+, d_{iU}^- (i = 1,2,\ldots,(n_1+2))$, for $d_{kL}^-, d_{kU}^+, d_{kL}^+, d_{kU}^- (k = 1,2)$, and $n_1$, the parts or elements of each of the $d_L^-, d_U^+, d_L^+, d_U^-$, and $(w_{iL}^-, w_{iU}^+, w_{iL}^+, w_{iU}^-) > 0$ with $\sum_i (w_{iL}^- + w_{iU}^+ + w_{iL}^+ + w_{iU}^-) = 1$ represent the numerical values that indicate the significance of reaching the goals within their corresponding target ranges, and $0 < \lambda < 1$; $\wedge$ stands for *min* operator.

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It is essential to note that the function Z in equation (15) is non-convex, meaning it does not have a single, straightforward shape and may have multiple local minima or maxima, making optimization more complex and typical characteristic of combinational optimization problems. To solve this, the mixed 0-1 programming approach [10], which is popular and relatively simple, is employed here.

### 5.1. Transformation of GP Model into Mixed 0–1 Programming Form

For $z_i \in \{0,1\}$ , (either 0 or 1), $(k = 1,2,....,(n_1 + 2))$, the regret function Z in (15) can be recast as :

$$\text{Minimize } Z = \lambda\left\{\sum_{i=1}^{n_1+2}(w_{iL}^{-}d_{iL}^{-} + w_{iU}^{+}d_{iU}^{+})z_i + \sum_{i=1}^{n_1+2}(w_{iL}^{+}d_{iL}^{+} + w_{iU}^{-}d_{iU}^{-})(1-z_i)\right\} + (1-\lambda)V, \tag{16}$$

$$\text{where}\left\{\max_{i \in n_1+2}\{(d_{iL}^{-} + d_{iU}^{+}) \wedge (d_{iL}^{+} + d_{iU}^{-})\}\right\} = V \tag{17}$$

Now,

$$(d_{iL}^{-} + d_{iU}^{+}) \wedge (d_{iL}^{+} + d_{iU}^{-}) \leq V \text{ , } i = 1, 2,..,(n_1+2) \tag{18}$$

Next, by including the variable $z_i$ introduced earlier, the equivalent equation form as:

$$(d_{iL}^{-} + d_{iU}^{+})z_i + (d_{iL}^{+} + d_{iU}^{-})(1-z_i) \leq V,$$

$$\text{where } z_i \in \{0,1\} \text{ ; } i = 1, 2,..,(n_1+2) \tag{19}$$

The executable GP model is ultimately represented as follows:

$$\text{Minimize} Z = \lambda\left\{\sum_{i=1}^{n_1+2}(w_{iL}^{-}d_{iL}^{-} + w_{iU}^{+}d_{iU}^{+})z_i + \sum_{i=1}^{n_1+2}(w_{iL}^{+}d_{iL}^{+} + w_{iU}^{-}d_{iU}^{-})(1-z_i)\right\} + (1-\lambda)V, \tag{20}$$

in condition of equations (15) and (19)

Here, the most fit function is:

eval ($V_P$) = $(Z)_P$ , P = 1,2,…, pop_size.

The best chromosome $V^*$ with highest fitness score at a generation is found below:

$$V^* = \min\{\text{eval}(V_P) \mid P = 1,2....,\text{pop\_size}\}.$$

A numerical example is solved to demonstrate the proposed method

Under the constraint sets specified in (15) and (19), the genetic algorithm (GA), which serves primarily as a satisficing decision-maker rather than a strict optimizing decision maker, can effectively be applied to minimize the regret function Z in (20). This strategy greatly assists in achieving a mutually satisfactory decision through the reduction of regrets for both decision makers.

the candidate with the smallest fitness value, expressed as

eval ($V_P$) = $(Z)_P$ , P = 1,2,…, pop_size.

To show case the proposed method, a comprehensive example is presented for illustration.

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## 6. ILLUSTRATION THROUGH EXAMPLE

Suppose the two variables involved in the decision-making process, $x_1$ and $x_2$, are controlled by the leader and the follower, respectively.
In that case, the FBLPP with interval coefficients can be expressed as:

Find X ($x_1$, $x_2$) so as to

$$\underset{x_1}{\mathrm{Max}}\, F_1(x_1, x_2) = \frac{[1,2]x_1 + [5,11]x_2 + [7,8]}{[4,5]x_1 + [3,7]x_2 + [3,3]}, \quad \text{(upper hierarchical problem)} \tag{21}$$

and,

$$\underset{x_2}{\mathrm{Max}}\, F_2(x_1, x_2) = \frac{[3,4]x_1 + [1,2]x_2}{[6,7]x_1 + [2,4]x_2 + [5,6]}, \quad \text{(lower hierarchical problem)} \tag{22}$$

With condition

$$\begin{aligned} &2x_1 + x_2 \le 7, \quad -2x_1 + 4x_2 \le 9, \\ &5x_1 + 2x_2 \ge 6, \quad x_1 \le 2, \\ &x_1, x_2 \ge 0. \end{aligned} \tag{23}$$

Based on the procedure, the Leader's goal in interval valued form is given as

$$\left(\frac{x_1 + 5x_2 + 7}{5x_1 + 7x_2 + 3}, \frac{2x_1 + 11x_2 + 8}{4x_1 + 3x_2 + 3}\right),$$

For the 2nd decision maker that follows the first is:

$$\left(\frac{3x_1 + x_2}{7x_1 + 4x_2 + 6}, \frac{4x_1 + 2x_2}{6x_1 + 2x_2 + 5}\right).$$

The solution by using the given scheme by the following the GA parameters proved effective during the solution determination:

Crossover probability $P_c = 0.8$, mutation probability $P_m = 0.08$, population size of 100, and chromosome length of 30.

The GA was programmed in the C language and executed on an Intel Pentium IV processor running at 2.66 GHz with 1 GB of RAM.

The first level decision maker's highest and lowest solutions are determined as: $(X_1^{lb}, X_2^{lb}; T_{1U}^*) = (0, 3; 3.41)$

and $(X_1^{lw}, X_2^{lw}; T_{1L}^*) = (4, 0; 0.47)$, respectively.

The follower's optimal and least favorable solutions have been determined. These represent the best and worst outcomes for the follower in the decision context.

$$(X_1^{fb}, X_2^{fb}; T_{2U}^*) = (1.5, 4.5; 0.65)$$

and $(X_1^{fw}, X_2^{fw}; T_{2L}^*) = (0, 1; 0.1)$, respectively.

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Then, the goals in the conventional GP formulation can be derived:

$$\left(\frac{x_1+5x_2+7}{5x_1+7x_2+3}, \frac{2x_1+11x_2+8}{4x_1+3x_2+3}\right) = [0.47, 3.41] \ ,$$

$$\left(\frac{3x_1+x_2}{7x_1+4x_2+6}, \frac{4x_1+2x_2}{6x_1+2x_2+5}\right) = [0.1, 0.65] \ ,$$

Once again, the decision variable $x_1$ with its corresponding target interval is given as

$$[1, 1]\ x_1 = [0, 1.5].$$

Then, the goals in standard GP formulation are obtained as

$$\frac{x_1+5x_2+7}{5x_1+7x_2+3} + d_{1L}^- - d_{1L}^+ = 0.47, \qquad \frac{2x_1+11x_2+8}{4x_1+3x_2+3} + d_{1U}^- - d_{1U}^+ = 3.41,$$

$$\frac{3x_1+x_2}{7x_1+4x_2+6} + d_{2L}^- - d_{2L}^+ = 0.1, \qquad \frac{4x_1+2x_2}{6x_1+2x_2+5} + d_{2U}^- - d_{2U}^+ = 0.65,$$

$$x_1 + d_{3L}^- - d_{3L}^+ = 0,$$

$$x_1 + d_{3U}^- - d_{3U}^+ = 1.5 \qquad (24)$$

By applying the expressions of Z in equation (20) and following the outlined procedure, the executable GP model in the form of a mixed integer (0-1) programming problem is formulated as follows:

To find $X(x_1, x_2)$ such that

$$\text{Minimize } Z = \lambda\left\{\sum_{i=1}^{3}\{(w_{iL}^- d_{iL}^- + w_{iU}^+ d_{iU}^+) z_i + (w_{iL}^+ d_{iL}^+ + w_{iU}^- d_{iU}^-)(1-z_i)\}\right\} + (1-\lambda)V, \qquad (25)$$

to satisfy the goal constraints in (24)

subject to, $(d_{iL}^- + d_{iU}^+)z_i + (d_{iL}^+ + d_{iU}^-)(1-z_i) \le V$, i=1,2,3 ; $z_i \in \{0,1\}$, and the constraints in (23).

We assume equal weights $w_1 = w_2 = w_3 = 1/3$ that are assigned for achieving the goals. The problem is then solved using the GA method, with the function Z from (25) serving as the fitness function.

The decision outcome is obtained as:

$$(x_1, x_2) = (0, 2.0656).$$

The objective function values achieved, given in interval form, are:

$$Z_1 = [0.99,\ 3.34] \quad \text{and} \quad Z_2 = [0.14,\ 0.45],$$

The results indicate a satisfactory decision is achieved by appropriately allocating decision-making powers between both decision makers.

It should be noted that if the objectives had crisp coefficients instead of interval ones, the problem could be easily solved using the mid-point arithmetic method [14] within the proposed framework.

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Additionally, the proposed GA-based approach avoids the computational difficulties caused by fractional objectives [6] and the heavy processing burden associated with traditional linearization methods [21].

## 7. CONCLUSION

The principal establishment of the proposed IP method for addressing the fractional bilevel situation lies in its ability to circumvent the ambiguity often introduced by fixed objective values in conventional methodologies. Instead, it incorporates goal values defined within flexible interval ranges, thereby accommodating the diverse requirements and preferences of DMs more effectively.

This approach enables the attainment of objective values within specified bounds, which can dynamically vary depending on the input parameters. These intervals offer adaptability and can be fine-tuned to align with the strategic priorities and operational constraints of an organization, particularly within hierarchical decision-making frameworks [33, 34].

Moreover, the proposed method holds potential for scalability and can be extended to tackle multi-objective optimization problems in complex, large-scale hierarchical organizations. Such extensions present promising avenues for future research.

In summary, this innovative approach not only enhances the modeling of practical hierarchical decision-making problems but also contributes to the development of robust, adaptive strategies that support the sustainable advancement of organizations in an increasingly competitive and uncertain global environment.

## REFERENCES

1. [1] Ignizio J.P. (1976). *Goal Programming and Extensions,* Lexington, Massachusetts, D.C. Health.
2. [2]. Bard J.F and . Falk J.E. (1982) "An explicit solution to multi-level programming problem", *Computers and Operations Research*, vol. 9, pp.77-100.
3. [3]. Bialas W.F., and Karwan M.H. (1982). "On two level optimization", *IEEE Transactions on Automatic Control*, vol. 27, pp. 211-214.
4. [4] Candler W., and Townsley R. (1982). "A linear two-level programming problem",*Computers and Operations Research*, vol.9, pp.59-76,
5. [5] Bialas W.F., and Karwan M.H. (1984). "Two level linear programming", *Management Sciences*, vol.30, pp. 1004-1020.
6. [6] Ludhandjula M. K (1984). "Fuzzy approaches for multiple objectives linear fractional optimization", *Fuzzy Sets and Systems*, vol. 13, pp. 11-23.
7. [7] Zimmermann H.-J. (1987). *Fuzzy Sets, Decision Making and Expert Systems*, Kluwer Academic Publisher, Boston, Dordrecht, Lancaste.
8. [8] Anandalingam G. (1988). "A mathematical programming modelof decentralized multi-level system", *Journal of Operational Research Society*, vol. 39(11), pp.1021-1033.
9. [9] Goldberg D.E. (1989). *Genetic Algorithms in Search, Optimization, and Machine Learning*, Addison-Wesley, Reading. MA.

ISSN No: 2350-1278
Peer Reviewed & Refereed Journal (IF: 7.9)
Journal Website www.nrjitis.in

10. [10] Inuiguchi M., and Kume Y. (1991). "Goal Programming Problems with interval coefficients and target intervals", *European Journal of Operational Research*, vol.52, pp.345-360.

11. Romero C. (1991). *Handbook of critical issues in goal programming*, Pergamon Publ. Corp.

12. [12] Mathiew R,, Pittard L., and Anandalingam G. (1994). "Genetic Algorithm based method for bilevel linear programming", *Recherch Ope'rationnelle / Operations Research*, vol. 28(1), pp. 1-21.

13. [13] Gen M., Liu B., and Ida K. (1996). "Evolution program for deterministic and stochastic optimizations", *European Journal of Operations Research*, vol. 94, pp. 618-625.

14. [14] Moore R.E., (1996). *Interval Analysis*, Prentice Hall, New Jersey.

15. [15] Sakawa M., Nishizaki I., and Hitaka M. (1999) "Interactive fuzzy programming for multi-level 0-1 programming problems through genetic algorithms", *European Journal of Operational Research*, vol. 114, pp.580-588.

16. [16] Sakawa. M., Nishizaki I ., and Hitaka M. (2001). "Interactive fuzzy programming for multi-level 0-1 programming problems with fuzzy parameters through genetic algorithms", *Fuzzy Sets and Systems*, vol. 117, pp.95-111.

17. [17] Deb K. (2002). *Multi-objective Optimization using Evolutionary Algorithms*, John Wiley & Sons, Ltd.

18. [18] Hejazi S.R., Memariani A., Jahanshahloo G., and Sepehri M.M .(2002). "Linear bilevel programming solution by genetic algorithm", *Computers and Operations Research*, vol. 29, pp.1913-1925.

19. [19] Liu B. (2002). *Theory and Practice of Uncertain Programming*, 1st ed., Physica-Verlag, Heidelberg,.

20. [20] Pal B.B., and Moitro B. N. (2003). (2003) . "Fuzzy goal programming procedure for solving quadratic bilevel programming problems", *International Journal of Intelligent Systems*, vol.18 (5), pp. 529-540.

21. [21] Pal B.B., Moitro B. N., and U. Maulik. (2003). "A Goal Programming Procedure for Fuzzy Multiobjective Linear Fractional Programming Problem". *Fuzzy Sets and Systems*, vol. 139, pp. 395 – 405.

22. [22] Pal B. B., and Pal R. (2003). "A linear multilevel programming method based on fuzzy programming", *Proceedings of APORS 2003*, vol. 1, pp.58-65.

*23.* [23] Maside X., Lee A.W., and Charlesworth, B. (2004). "Selection on Codon Usage in *Drosophila Americana"*, Current Biology, vol. 14, pp.150-154.

24. [24] Biswas A. , and Pal B.B. (2007) "Fuzzy goal programming procedure for multiobjective bilevel programming problems", *International Journal of Computer, Mathematical Sciences and Appl.*, vol. 1 , No. 1, pp. 87-98.

**25.** [25] Olivera C., and Antunes C.H. (2007). "Multiple objective Linear Programming Models with Interval Coefficients – an illustrated overview", *European Journal Of Operational Research,* vol. 181, pp. 1434-1463.

26. [26] Pal B.B., and Gupta, S.S. (2008). "A Goal Programming approach for solving interval valued Multiobjective Fractional Programming Problems by using Genetic Algorithm", *Proceedings Of IEEE 10th colloquium and International Conference on Industrial and Information Science 2008 (ICIIS 2008),* pp.1-6.

27. [27] Pal B.B., and Sen S. (2008). "A Goal Programming Procedure for solving Interval valued Multiobjective Fractional Programming Problems", *Proceedings of Sixteenth Int. Conf. on Advanced Computing and Communication 2008 (ADCOM 2008),* pp. 297-302.

28. [28] Yang Z., and Nielsen, R.(2008). "Mutation-Selection Models of Codon Substitution and Their Use to Estimate Selective Strengths on Codon Usage", *Mol. Biol. Evol*., vol. 25(3), pp. 568–579.

29. [29] Pal B.B., and Gupta, S.S. (2009). "A Genetic Algorithm Based Fuzzy Goal Programming Approach for solving Fractional Bilevel Programming Problems", *Proceedings of International Conference of Operations Research in Engineering and Management (ICOREM 2009)*, pp. 306-325.

30. [30] Pal B. B., Gupta S.S., and . Chakraborti D. (2009). "A Fuzzy Goal Programming Procedure for Solving Linear Multilevel Programming Problems via Genetic Algorithm", *Proceedings of International Conference of Operations Research in Engineering and Management (ICOREM 2009)*, pp. 355-373.

31. [31] Pal B.B., Sen S., and Kumar M. (2009) "A Goal Programming Method for Solving Academic Personnel Planning Problems with Interval-Valued Resource Goals in University Management System", *Proceedings of International Conference of Operations Research in Engineering and Management (ICOREM 2009)*, pp. 377-399.

32. [32] Chakraborti D. (2015). "Evolutionary technique based goal programming approach to chance constrained interval valued bilevel programming problems", OPSEARCH, pp. 1-19, 2015, ISSN No.: 0030-3887. Doi: 10.1007/s12597-015-0238-1.

33. [33] Chakraborti D. (2011). "A Fuzzy Goal Programming Based Stochastic Simulation Approach to Chance Constrained Bilevel Programming Problems via Genetic Algorithm", Survey, Vol. 54, Nos. 1&2, 2014, ISSN No.: 0586-0008.

34. [34] Chakraborti D. (2018). "Mathematical Modelling of Land Allocation Problem for Optimal Cropping Plan in Agricultural System Using Hybrid Goal Programming", International Journal of Engineering, Science and Research, Volume 13, Number 4, pp. 1-15, ISSN: 2320-0294.